AP Calculus AB

WS 79 - The Second Derivative Test

1) $f(x) = x^3 - 12x^2 + 45x$

$f'(x) = 3x^2 - 24x + 45 = 0$

$x^2 - 8x + 15 = 0$

$(x-5)(x-3) = 0$

$x = 5 \quad x = 3$

$f''(x) = 6x - 24$

$f''(5) = 6 > 0 \quad f''(3) = -6 < 0$

* $f(x)$ has a local min @ $x = 5$ b/c $f'(5) = 0$ & $f''(5) > 0$
* $f(x)$ has a local max @ $x = 3$ b/c $f'(3) = 0$ & $f''(3) < 0$

2) $f(x) = x^4 - 8x^2 + 1$

$f'(x) = 4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0 \quad x = 2 \quad x = -2$

$f''(x) = 12x^2 - 16$

$f''(0) = -16 < 0 \quad f''(2) > 0 \quad f''(-2) > 0$

* $f(x)$ has a local max @ $x = 0$ b/c $f'(0) = 0$ & $f''(0) < 0$
* $f(x)$ has a local min @ $x = -2, 2$ b/c $f'(2) = f'(2) = 0$ & $f''(2) = f''(-2) > 0$

3) $f(x) = \sin^2 x + \cos x \quad [0, \pi]$

$f'(x) = 2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \quad 2\cos x - 1 = 0$

$x = 0 \quad \cos x = 1/2$

$x = \pi/3$

$f''(x) = 2\sin x(-\sin x) + 2\cos x \cos x - \cos x$

$= -2\sin^2 x + 2\cos^2 x - \cos x$

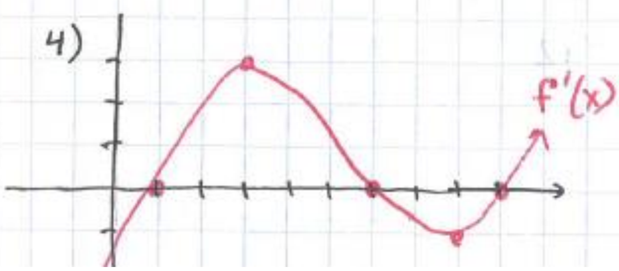
$f''(0) = 1 > 0 \quad f''(\frac{\pi}{3}) = -2\left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{1}{2}\right)^2 - \frac{1}{2}$

$= -3 + \frac{1}{2} - \frac{1}{2}$

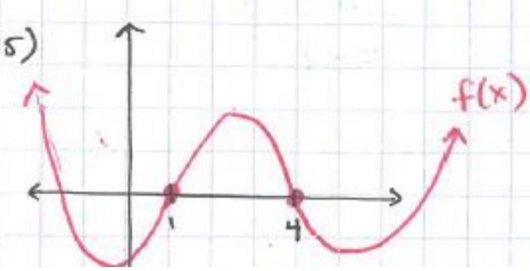
$= -3 < 0$

* $f(x)$ has a local min @ $x = \frac{\pi}{2}$ b/c $f'(\frac{\pi}{3}) = 0$ & $f''(\frac{\pi}{2}) > 0$
* $f(x)$ has a local min @ $x = \frac{\pi}{3}$ b/c $f'(\frac{\pi}{3}) = 0$ & $f''(\frac{\pi}{3}) < 0$.

4)



5)



) $f(x) = x^4$
$f'(x) = 4x^3$
$f''(x) = 12x^2$
$f''(x)$ does not change signs $\rightarrow$ f has no P.o.I.

FREE RESPONSE QUESTION

a) $f(x)$ has a local max @ $x = -2$ b/c $f'(x)$ Δs signs from + to −.

b) $f(x)$ is dec on $(-2, 4)$ b/c $f'(x) < 0$

$f(x)$ is concave down on $(-3, -1)$ & $(1, 3)$ b/c $f''(x) < 0$
(f' is dec)

$\therefore$ $f(x)$ is both dec & concave down on $(-2, -1)$ & $(1, 3)$.

c) $f(x)$ has P.o.I. @ $x = -1, 1, 3$, b/c f'' changes signs

d) $f(x) = 3 + \int_1^x f'(t)\,dt$

$f(4) = 3 + \int_1^4 f'(t)\,dt$

$= 3 + -12$

$= -9$

$f(-2) = 3 + \int_1^{-2} f'(t)\,dt$

$= 3 + 9$

$= 12$